k^2 in the expansion of $\pi K'/K$. The coefficients of degree, $8(n-3)$, $8(n-2)$, $8(n-1)$, and $8n$, are determined by the coefficients of degree $\leqslant 2n$. That this relationship provides of convincing check on the accuracy of the expansion of $\pi K'/K$, in powers of k^2, is shown in the Appendix.

IV. Extensions

A similar argument applies to the case considered by Oberhettinger and Magnus [1, p. 61] when the strip inner conductor is centrally located. Now we determine the capacitance of the line segment *ea* with respect to the infinite line segment *bd* in the *t* plane of Fig. 1. Using (4), it is found that k is given by

$$k^2 = \frac{8\delta(1+\delta^2)}{(1+\delta)^4}. \tag{17}$$

This differs from (6) only in that δ^2 has been replaced with δ. It follows then from (16) that, for this case,

$$\frac{K'(k)}{K(k)} = \frac{1}{4}\frac{K'(\delta^2)}{K(\delta^2)}. \tag{18}$$

Finally

$$C_0 = 2\frac{K(k)}{K'(k)} = 8\frac{K(\delta^2)}{K'(\delta^2)}. \tag{19}$$

The reader will find it interesting to compare this solution with that given by Nehari [4, p. 293].

The capacitance of a line segment, one of whose ends falls on the origin, is given by the same expansion. We are now concerned with the capacitance in the upper half t plane between the line segment *ea* and the infinite line segment *cd*. For this case, using (4),

$$k^2 = \frac{\delta}{(1+\delta)^2}. \tag{20}$$

It follows that

$$\frac{K'(k)}{K(k)} = \frac{1}{2}\frac{K'(\delta)}{K(\delta)} \tag{21}$$

and

$$C_0 = 4\frac{K(\delta)}{K'(\delta)}. \tag{22}$$

Appendix

Determining (9) from the power series for k^2 required additional terms in the expansion for $K(k)/K'(k)$ given in [3, p. 1219]. These terms are

$$\frac{K'(k)}{K(k)} = \frac{1}{2}\cdots + \frac{1057889591339}{26388279066624}k^{24} + \frac{2069879045935}{57174604644352}k^{26} + \frac{32456762953369}{985162418487296}k^{28} + \frac{63713529525287}{2111062325329920}k^{30} + \frac{512963507737401997}{18446744073709551616}k^{32} + \cdots. \tag{A1}$$

The coefficients in (9) were obtained by using all the coefficients in (A1), including of course those of [3, p. 1219], in (6), (7), and (8). The fact that the coefficients in (9) are known to be correct from (16) is a very convincing indication of the accuracy of (A1). Moreover, it has been found that changing the value of the coefficient of k^{32} in (A1) by only one digit in the last place will alter the coefficient of δ^{32} in (9).

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Equivalent Circuits for Dielectric Posts in a Rectangular Waveguide

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***Abstract*— Scattering by dielectric posts in a rectangular waveguide is investigated by a combination of the finite and boundary element methods (CFBEM), and the equivalent circuits are derived. Some of the lossless dielectric post resonances in a rectangular waveguide can be physically realized by a lattice circuit, and the interaction between two posts can be evaluated by this circuit.**

I. Introduction

Recently, studies on scattering in waveguides containing dielectric posts have been reported [1]–[10]. Dielectric resonators are simple and small in size and ceramic dielectrics are readily available, so that ceramic dielectrics with high relative permittivity and temperature stability have been used to design microwave filters. Sahalos and Vafiadis [4] have considered the design of bandpass or band-stop filters, and Gesche and Löchel [7] a tunable band-stop filter. Hsu and Auda [9] have physically realized lossy dielectric post resonance by a lumped network. We expect that an equivalent circuit which can represent the dielectric resonance in a rectangular waveguide would be useful for the design of microwave filters.

We show that some of the lossless dielectric post resonances in a rectangular waveguide can be physically realized by the lumped networks. The interaction between two posts in a rectangular waveguide can be evaluated by these lumped networks.

II. Methods

The problems are analyzed by a combination of the finite and the boundary element method (CFBEM) [10].

The equivalent circuit for a post in a rectangular waveguide is shown in terms of two-port networks. The T network is commonly used. However, it is found that the shunt arm reactance of the T network decreases with increasing frequency [5], [9]. In the case where the post structures are symmetrical about some plane perpendicular to the axis of the transmission line, we can use the lattice network [9], [13]. It has been shown in [13] that any two-port network is physically realizable in the lattice form; i.e., it is unnecessary to use any negative inductances or capacitances to construct the lattice.

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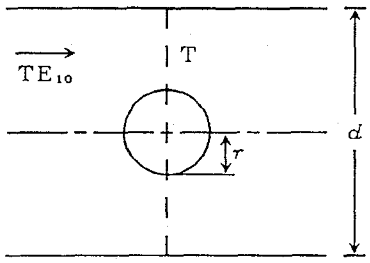

Fig. 1. Centered dielectric post in a rectangular waveguide.

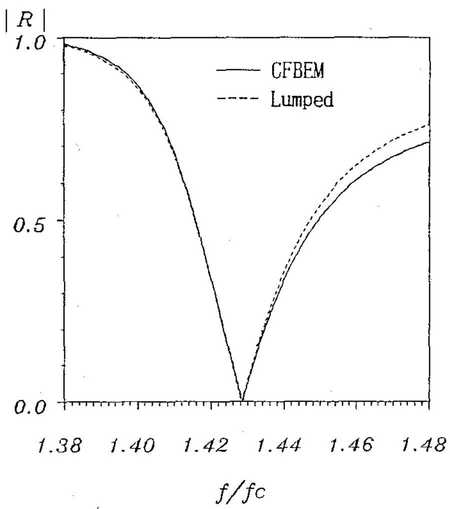

Fig. 2. Magnitude of reflection coefficient versus f/f_c for a waveguide loaded with a centered post of $r/d = 0.05$ with $\epsilon = 112.5$.

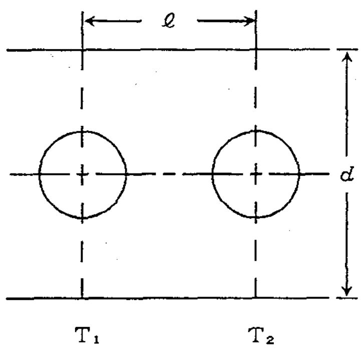

Fig. 3. Rectangular waveguide loaded with two centered posts.

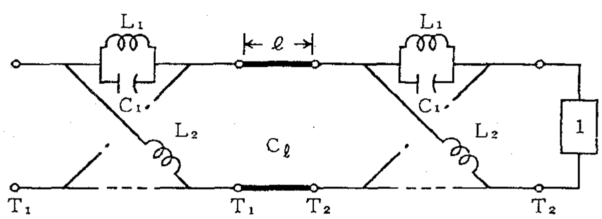

Fig. 4. A chain of equivalent circuits connected in cascade.

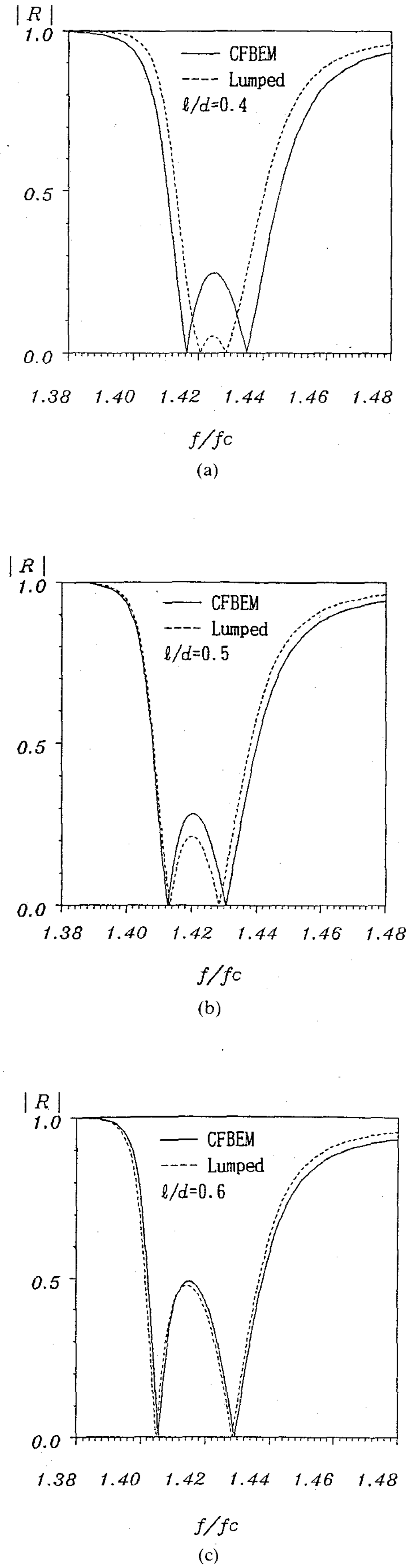

Fig. 5. Magnitudes of reflection coefficient versus f/f_c with various l/d values for the waveguide loaded with two centered posts of $r/d = 0.05$ with $\epsilon = 112.5$. (a) $l/d = 0.4$. (b) $l/d = 0.5$. (c) $l/d = 0.6$.

Using the reflection and the transmission coefficients at the reference plane T shown in Fig. 1, the equivalent circuit parameters of the T and lattice networks are represented by only reactances, since a dielectric post is assumed to be lossless.

III. Numerical Results

We take the example given by Marcuvitz [1], Nielsen [2], Araneta *et al.* [3], Sahalos and Vafiadis [4], Hsu and Auda [5], and Leviatan and Sheaffer [6], which computes the scattering parameters and equivalent network elements as a function of relative permittivity ϵ for a centered lossless dielectric post of $r/d = 0.05$ at $\lambda/d = 1.4$, where d is the width of the rectangular waveguide, r is the radius of the cylindrical post as shown in Fig. 1, and λ is the wavelength in free space. The results obtained agree well with Leviatan's [6]. The resonance occurs at $\epsilon = 112.5$.

Now we consider the bandpass filter which takes a minimum of the reflection coefficient R at $\lambda/d = 1.4$ for a centered lossless dielectric post of $r/d = 0.05$ and with $\epsilon = 112.5$. Fig. 2 shows the magnitude of the reflection coefficient R by the solid line, where the frequency is normalized by the cutoff frequency f_c. It is found in this case that the shunt reactance for the T network decreases with increasing frequency, while the series and cross reactances for the lattice network increase with increasing frequency. The series arm resonance occurs at $f_0/f_c = 1.42089$.

We consider the equivalent lattice circuit, where the series reactance corresponds to a resonant parallel LC network and the cross reactance to an inductor. By the relationship between the original and equivalent circuit reactances, the normalized lumped lattice circuit is obtained as

$$L_1\omega_c = 0.0233 \qquad C_1\omega_c = 21.26 \qquad \text{and} \qquad L_2\omega_c = 0.227.$$

L_1 and C_1 are the inductor and the capacitor for the parallel LC network, respectively, and L_2 is the inductor for the cross arm.

The magnitude of the reflection coefficient R calculated by the lumped lattice circuit is shown by the broken line in Fig. 2, and it agrees well with $|R|$ obtained by the CFBEM.

Next we consider the waveguide loaded with two posts shown in Fig. 3, where each post is the same as the previous one and is located in the middle of the waveguide. If there is no interaction between the posts, the equivalent circuit of the waveguide may be assumed to be a chain of previous equivalent circuits connected in cascade as shown in Fig. 4. C_l is the transmission line of length l.

Fig. 5(a), (b), and (c) shows the magnitudes of the reflection coefficient R versus f/f_c with various l/d values, where the solid and broken lines correspond to the results obtained by the CFBEM and by the equivalent circuit cascaded by previous ones, respectively. It can be seen that the two agree well at $l/d \geqslant 0.5$. In the frequency range under consideration, i.e., $1.38 \leqslant f/f_c \leqslant 1.48$, the wavelength in the waveguide λ_g is $0.5258d \geqslant \lambda_g/4 \geqslant 0.4583d$. For this example, the reflection coefficient R for the waveguide loaded with two centered posts shown in Fig. 3 seems to be evaluated fairly well at $l > \lambda_g/4$ by the equivalent circuit shown in Fig. 4. For l/d less than 0.5, disagreement occurs because the coupling between the posts was neglected in the circuit analysis.

IV. Conclusions

We show that some of the lossless dielectric post resonances in a rectangular waveguide can be physically realized by a lattice circuit and that the interaction between two posts in a rectangular waveguide can also be evaluated by the lattice circuit.

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A Statistical Method for Calibrating the Six-Port Reflectometer Using Nonideal Standards

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***Abstract*—This paper presents an alternative method for calibrating the six-port reflectometer. Through the use of a redundant set of calibration standards, an estimate of the 11 real calibration constants is determined in the minimum-mean-squared-error sense. This technique enables the user to weight the contribution of each standard to the calibration process as a function of confidence in the quality of that standard. The resulting computer algorithm is quite straightforward and provides a direct measure of the tightness of fit between the estimated six-port model and the observed data.**

I. Introduction

Since their inception, six-port networks have found applications ranging from power meters to vector network analyzers. Because of the inherent simplicity and stability of the six-port network, these applications have also covered the spectrum of

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